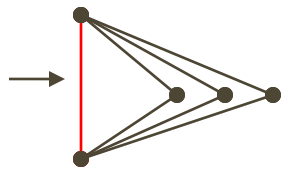


1

GWESP

How does the α decay parameter affect the value of the statistic?

Work through the calculation



$$gwesp = e^{\alpha} \sum_{i=1}^{n-2} \{1 - (1 - e^{-\alpha})^i\} sp_i \quad sp_i = \# \text{ of edges with } i \text{ shared partners}$$

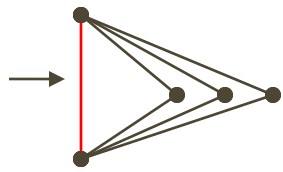
If we toggle the red edge on this will produce the following ESP statistics:

- 1 edge with 3 shared partners
- 6 edges with 1 shared partner

α	GWESP(α)
0	$e^0[(1 - (1 - e^{-0})^1) \times 6] + e^0[(1 - (1 - e^{-0})^3) \times 1] = 7$
0.5	$e^{0.5}[(1 - (1 - e^{-0.5})^1) \times 6] + e^{0.5}[(1 - (1 - e^{-0.5})^3) \times 1] = 7.55$
1	$e^1[(1 - (1 - e^{-1})^1) \times 6] + e^1[(1 - (1 - e^{-1})^3) \times 1] = 8.03$

The # of edges with 1+ shared partners

GWESP values for $\alpha = \{0 - 10\}$

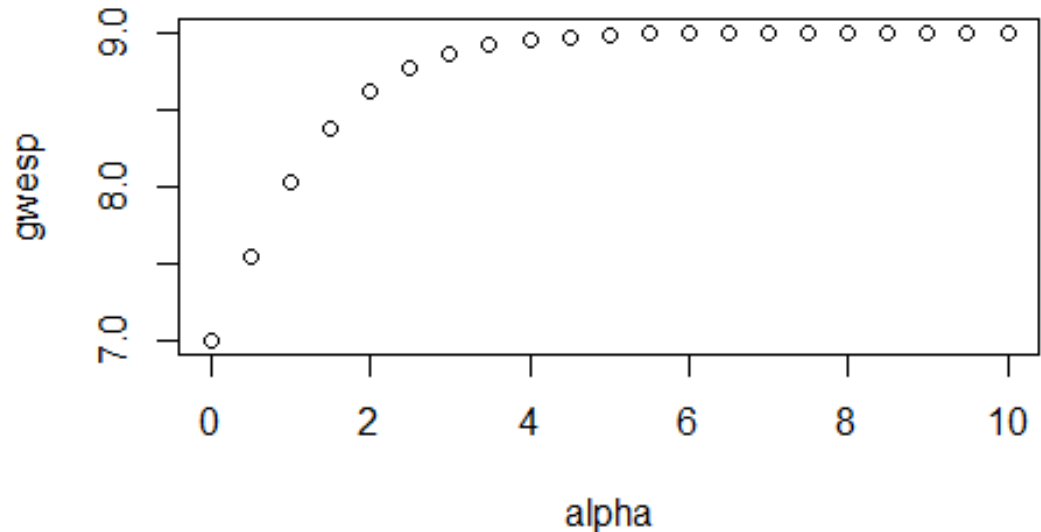


$$gwesp = e^{\alpha} \sum_{i=1}^{n-2} \{1 - (1 - e^{-\alpha})^i\} sp_i \quad sp_i = \# \text{ of edges with } i \text{ shared partners}$$

Count of edges in
each triangle
(i.e. # of triangles x 3)



Count of edges in at
least one triangle
(because only an edge's
first triangle counts)



Intuition

- When choosing a fixed value to test
 - $\text{gwesp}(0)$ – only the first shared partner contributes to the statistic value
 - Additional shared partners are ignored
 - $\text{gwesp}(4+)$ – asymptotes to the value of the triangle statistic
 - The term is no longer useful
 - In between [0+ to 3ish]
 - The value of the statistic rises quickly
 - The larger the value of α , the larger the value of the stat
 - When fixing α , in most cases we'll want a value < 1
- Remember that the gwesp statistic will be multiplied by its coefficient, θ , in the ERGM

References

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Hunter DR, Handcock MS. Inference in Curved Exponential Family Models for Networks. (2006) Journal of Computational and Graphical Statistics. 15(3):565-83. doi: 10.1198/106186006X133069.

Snijders, T.A.B., Pattison, P.E., Robins, G.L. and Handcock, M.S. (2006), New specifications for exponential random graph models. Sociological Methodology, 36: 99-153. <https://doi.org/10.1111/j.1467-9531.2006.00176.x>

For estimating α :

Jonathan Stewart, Michael Schweinberger, Michal Bojanowski, Martina Morris. Multilevel network data facilitate statistical inference for curved ERGMs with geometrically weighted terms. (2019) Social Networks 59: 98-119. <https://doi.org/10.1016/j.socnet.2018.11.003>

See also

Michael Levy's Shiny app for exploring the gwdegree term
<https://michaellevy.shinyapps.io/gwdegree/>

This is a really great app for understanding how the two parameters in a gw- term (decay and coefficient) influence the resulting degree or esp distribution. He focuses on the gwdegree term, but gwesp behaves similarly (at least with respect to the esp distribution; it has a different effect on network structure generally).