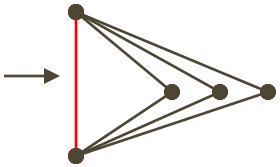
GWESP

How does the α decay parameter affect the value of the statistic?

Work through the calculation



$$gwesp = e^{\alpha} \sum_{i=1}^{n-2} \{1 - (1 - e^{-\alpha})^i\} sp_i$$

sp_i = # of edges with i shared partners

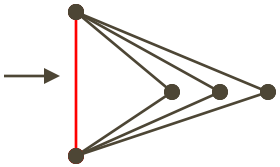
If we toggle the red edge on this will produce the following ESP statistics:

- 1 edge with 3 shared partners
- 6 edges with 1 shared partner

α	GWESP(α)	
0	$e^0\left[\left(1-\left(1-e^{-0}\right)^1\right)\times 6\right] + e^0\left[\left(1-\left(1-e^{-0}\right)^3\right)\times 1\right]$	= 7
0.5	$e^{0.5}\left[\left(1-\left(1-e^{-0.5}\right)^1\right)\times 6\right] + e^{0.5}\left[\left(1-\left(1-e^{-0.5}\right)^3\right)\times 1\right]$	= 7.55
1	$e^1[(1-(1-e^{-1})^1)\times 6] + e^1\left[\left(1-(1-e^{-1})^3\right)\times 1\right]$	= 8.03

The # of edges with 1+ shared partners

GWESP values for α = {0 - 10}

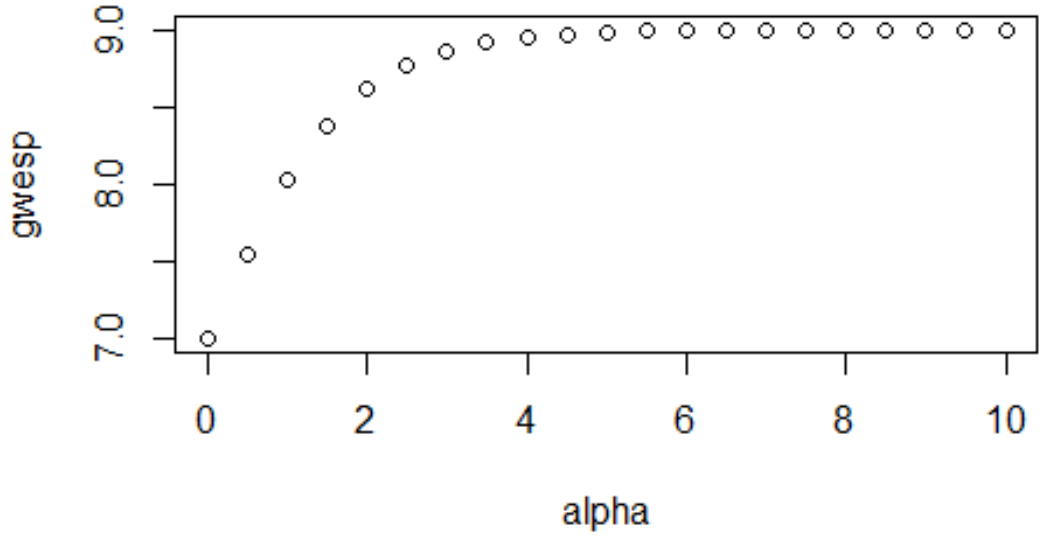


$$gwesp = e^{\alpha} \sum_{i=1}^{n-2} \{1 - (1 - e^{-\alpha})^{i}\} sp_i$$

sp_i = # of edges with i shared partners

Count of edges in each triangle (i.e. # of triangles *x* 3)

Count of edges in at least one triangle (because only an edge's first triangle counts)



Intuition

- When choosing a fixed value to test
  - gwesp(0) – only the first shared partner contributes to the statistic value
    - Additional shared partners are ignored
  - gwesp(4+) – asymptotes to the value of the triangle statistic
    - The term is no longer useful
  - In between [0+ to 3ish]
    - The value of the statistic rises quickly
    - The larger the value of alpha, the larger the value of the stat
  - When fixing α, in most cases we'll want a value < 1
- Remember that the gwesp statistic will be multiplied by its coefficient, θ, in the ERGM

References

Hunter DR. Curved Exponential Family Models for Social Networks. (2007) Social networks. 29(2):216-30. doi: 10.1016/j.socnet.2006.08.005. PubMed PMID: PMC2031865.

Hunter DR, Handcock MS. Inference in Curved Exponential Family Models for Networks. (2006) Journal of Computational and Graphical Statistics. 15(3):565-83. doi: 10.1198/106186006X133069.

Snijders, T.A.B., Pattison, P.E., Robins, G.L. and Handcock, M.S. (2006), New specifications for exponential random graph models. Sociological Methodology, 36: 99-153. https://doi.org/10.1111/j.1467-9531.2006.00176.x

For estimating α:

Jonathan Stewart, Michael Schweinberger, Michal Bojanowski, Martina Morris. Multilevel network data facilitate statistical inference for curved ERGMs with geometrically weighted terms. (2019) Social Networks 59: 98-119. https://doi.org/10.1016/j.socnet.2018.11.003

See also

Michael Levy's Shiny app for exploring the gwdegree term

https://michaellevy.shinyapps.io/gwdegree/

This is a really great app for understanding how the two parameters in a gw- term (decay and coefficient) influence the resulting degree or esp distribution. He focuses on the gwdegree term, but gwesp behaves similarly (at least with respect to the esp distribution; it has a different effect on network structure generally).