NME
2024
Network Modeling for Epidemics

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Model degeneracy

- What it is
- What it looks like
- What it represents
- How to avoid it

What it is

- Technical Definition:

  When a model places almost all probability on a small number of uninteresting graphs

- Most common “uninteresting” graphs:
  - Complete (all links exist)
  - Empty

- **Model degeneracy is a sign of misspecification**

  The model you specified would almost never produce the network you observed

What it looks like in ergm

- Your estimation will fail and you'll get an error like this

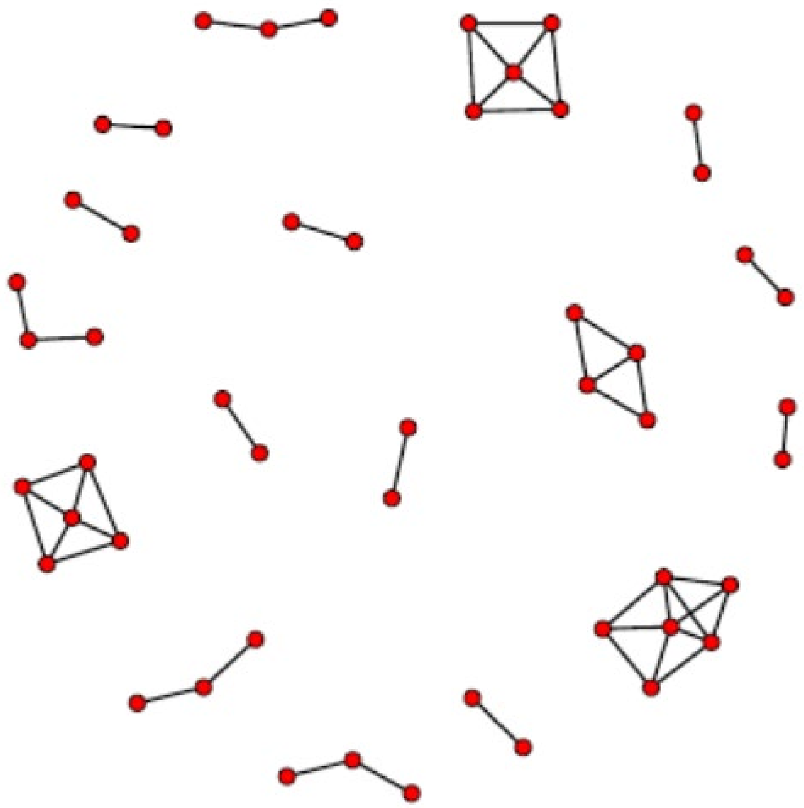
```
Error: Number of edges in a simulated network exceeds that in the observed by a factor of more than
20. This is a strong indicator of model degeneracy or a very poor starting parameter configuration.
If you are reasonably certain that neither of these is the case, increase the MCMLE.density.guard co
ntrol.ergm() parameter.
```

- What does this error message mean?

- When trying to fit this model, the algorithm heads off into networks that are much more dense than the observed network.

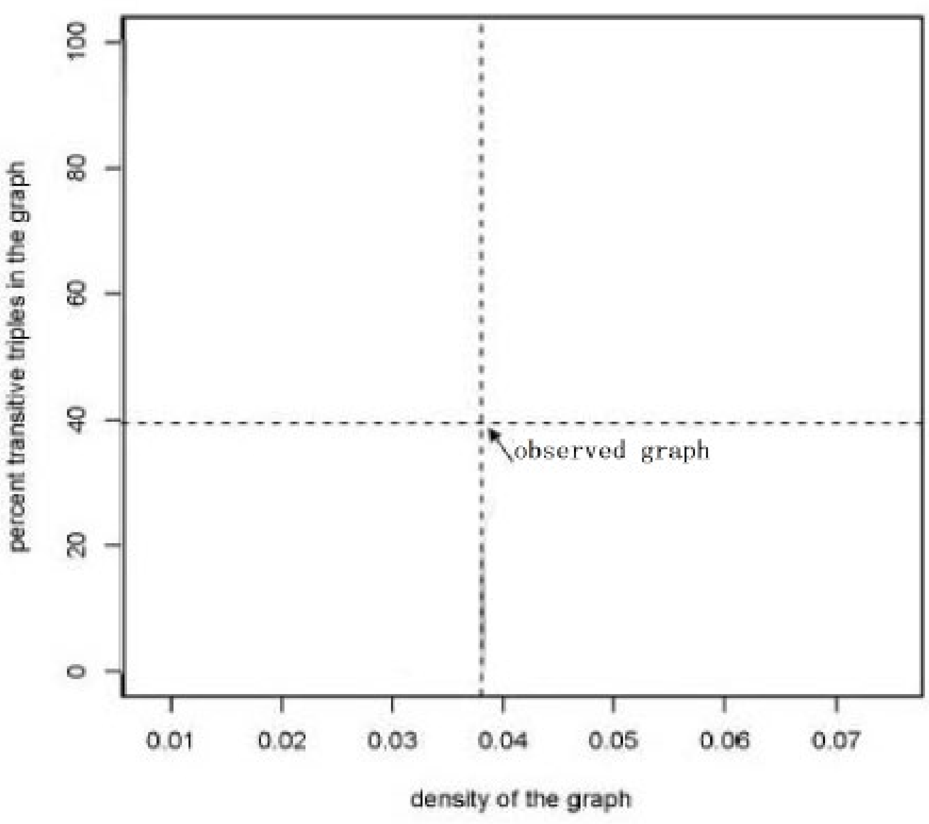
- Let's see why that is

Let’s take a simple example



- This network seems to have lots of triangles
  - 50 nodes
  - 4% density
  - 40% clustering
    - Fraction of all 2stars with the triangle completed
- So it would be natural to fit
  - edges + triangle model

Our network statistics



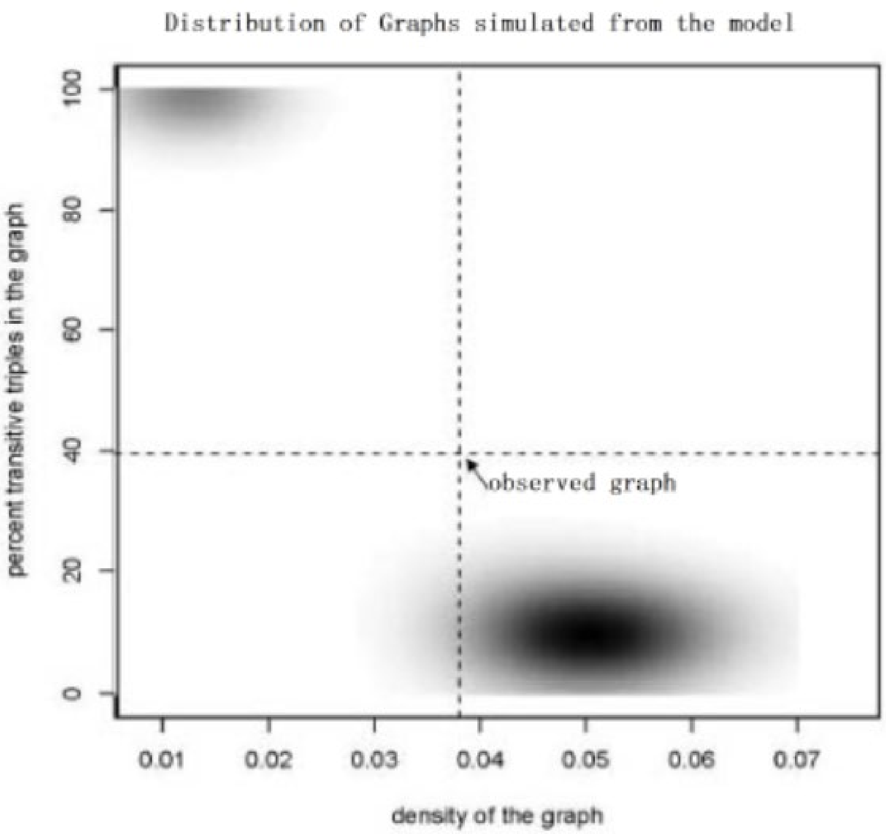
- We can represent our model statistics as a 2D plot

  And our observed graph in this plane

- Statistical theory guarantees that at the MLEs for θ:

  E(netstats) = Observed

At the MLE, this is what the model produces



- The theory is not wrong
- Indeed, the means of the netstats are correct
- But this model produces a *bimodal* distribution to get those means
- It would never produce the observed graph

MCMC Dx for a model like this

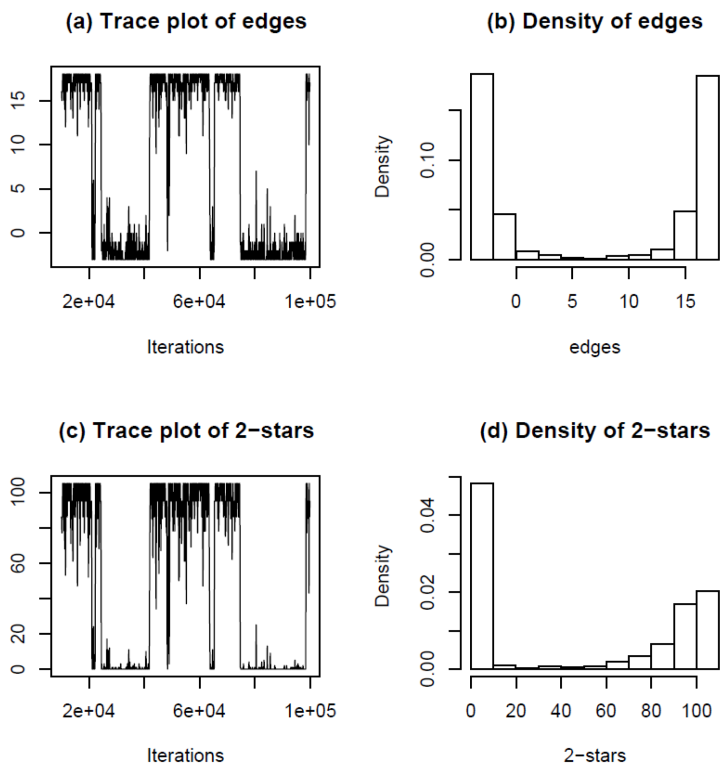


Figure 5: MCMC diagnostic plots for the model with $\theta = (-3.43, 0.683)$.

from Handcock 2003

- This is an example from fitting an edges+kstar(2) model
- If we let the MCMC iterate for a long time
- You can see the bimodal distribution in both the traceplots and the statistic densities

What this represents: a bad model

- The MCMC-MLE theory is fine, and there's nothing wrong with the algorithm

- The problem is the model

  The simple edges + triangle (or edges + kstar(2)) model would not produce our observed graph

- This is what model misspecification looks like with dependent data

Another way of thinking about this

- With a simple 2-parameter model, we can look at the networks produced (simulated) at all pairs of values of the coefficients
  - Ok, maybe not all, but many, many pairs

- Then answer the following questions:
  - How often does this model produce degenerate graphs?
  - How often does this model produce interesting graphs?

We already know it doesn't produce our network, but does it ever produce *ANY* networks that look reasonable?

And the answer is … **almost never**

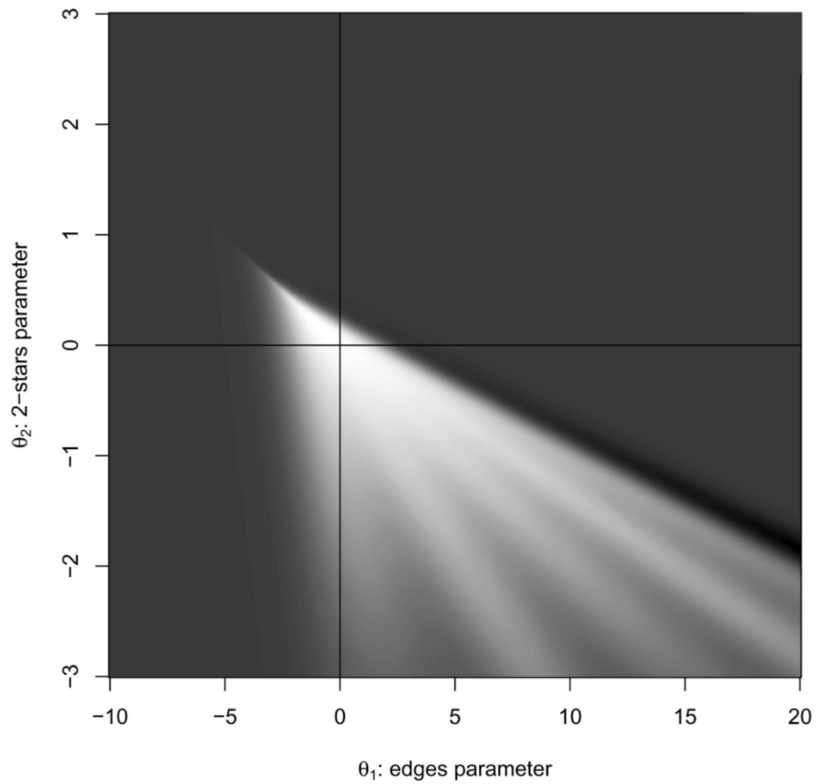


Figure 3: Cumulative Degeneracy Probabilities for graphs with 7 actors.

Graph from [Handcock 2003]

- This is the parameter space for an edges+kstar(2) model
- Shading indicates the frequency of reasonable networks
  - Black = none, all are degenerate
  - Gray = some
  - Light = more
- The only part of the parameter space with a high likelihood of a reasonable graph is close to the parameter set (0,0)
  - i.e., when there is no interesting structure to investigate

This is why we say this is a bad model

- These simple models with the homogeneous Markov graph statistics (k-stars and/or triangles) almost never produce interesting graphs

- So in general, it's best to avoid using these terms

- And instead use better specifications

Key references for model degeneracy

Handcock MS. (2003) Assessing Degeneracy in Statistical Models of Social Networks. CSSS working paper 39.

https://csss.uw.edu/node/4718

Schweinberger, M. (2011). Instability, Sensitivity, and Degeneracy of Discrete Exponential Families. *Journal of the American Statistical Association*, *106*(496), 1361–1370. https://doi.org/10.1198/jasa.2011.tm10747