

Network Modeling for Epidemics

1 Model specification

For dyad dependent models

SISMID: NME 2024

These models behave differently

- They're more like a complex system
 - And while the terms might look like they represent simple local configurations
 - E.g., Triangles and stars
 - They actually imply processes that cascade through the whole network
- Our intuition about them is often wrong
 - And that can lead to trouble

Simple example in statnetWeb

- Let's revisit the faux.mesa.high network
 - Recall that the CUG test showed there were many more triangles than expected for this level of tie density
 - How could you test this in an ERGM?

Network Plot	Attributes	Degree Distribution	Geodesic Distribution	More			
Null model tests ¥							
Model term			Number of simulations				
triangle		•	100		•	Run	
0 20 30 40 50 90 70							Observed value CUG distribution BRG distribution
-1	1 3 5						63

Triangle term

- The triangle term: $t(x) = \sum y_{ij} y_{jk} y_{ik}$
 - t(x) = # of triangles in the graph
 Here t(x) = 3 if the red edge is toggled on



This is one of the classic Markov Graph terms
 From the Frank and Strauss (1986) paper

Fit model: edges + triangle

statnetWeb Data	Network Descriptives Fit Model	MCMC Diagnostics	Goodness of Fit Simulations Help				
Network: faux.mesa.high	ERGM terms: Add Term(s) Reset Formula		Term Documentation Control Options Commonly used ergm terms Term cross-reference tables Select a term Compatible terms All terms Select or search for a term in the menu above. Image: Compatible terms	<i>←</i> →?			
Current ergm formula:	edges + triangle						
Summary statistics:	edges triangle 203 62						
Fit Model Save Current Model (0/5) Clear All Models							
Current Model Summary	Current Model Fit Report Model Co	mparison					

Error: Number of edges in a simulated network exceeds that in the observed by a factor of more than 20. This is a strong indicator of model degeneracy or a very poor starting paramete r configuration. If you are reasonably certain that neither of these is the case, increase the MCMLE.density.guard control.ergm() parameter.

▲ Download Summary (.txt)

What happened?

- The process triggered a built-in error detector
- And that automatically stopped the run
- Note the error message:

Number of edges in a simulated network exceeds that in the observed by a factor of more than 20.

The MCMC estimation chain was producing networks with WAY too many edges

To really see what's happening

- We need some advanced ergm options only available from the command line
- We will set some MCMC control parameters
 - To track each single toggle
 - And stop before triggering the built in error detector

If you want to try this yourself:

The MCMC dx plots ...



Why is this happening?

- Because this is a poorly specified model
 - It would never produce the network we observed
 - So the MCMC algorithm can't find ANY coefficients that work
 - And the ergm package automatically puts it out of its misery
- There's nothing wrong with
 - the theory
 - the algorithm
 - the data
- It's just a bad model

Intuition: Why is this a bad model?

Because triad formation doesn't actually work like this

The triangle term: $t(x) = \sum y_{ij} y_{jk} y_{ik}$



- With this term every additional triangle has the same impact, θ
 - So the odds of the red edge above are 3 times higher than an edge that creates only 1 triangle.
 - And an edge that creates 10 triangles has 10x higher odds
- This creates a cascading runaway process
 - Edges are most likely when they create huge clusters of triangles
 - And that's not what we see in our network

This is called "Model degeneracy"

- The model would not produce the observed network
 - Instead it places all probability on networks that are nearly empty, or nearly complete
 - On average, this gives the right value for the netstats, but you would never get the observed network from this model
- And this is what model misspecification looks like with dependent data:
 - You typically won't even get a fit to converge
 - So there's no fit object to diagnose
 - The classic diagnostic is the MCMC algorithm heading off into graphs with much higher density than observed
- See the appendix on Model Degeneracy for more details

The solution: Better specification

• New statistic:
$$gwesp = e^{\alpha} \sum_{i=1}^{n-2} \{1 - (1 - e^{-\alpha})^i\} sp_i$$



- gwesp = a weighted sum of the triangles created by each edge
- Where the weights decline for each additional triangle created
 - For each additional "shared partner" of an edge (like the red edge here)
 - This sets declining marginal returns, with a smooth decay function
- The decay function we use involves a geometric weighting
 - Hence the name: geometrically weighted edge-wise shared partners
 - a.k.a. GWESP

Details in the Appendix

Practical advice

- Stay away from the canonical Markov graph terms
 - Unless you are working with very small networks
- The ergm package includes both the Markov graph terms and more stable alternatives

To represent	Markov graph ergm term	More stable alternatives
Ties on a node	kstar	degree(n) (non-parametric) gwdegree (parametric)
Triads	triangle	esp(n) (non-parametric) gwesp (parametric)